OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Friday **14 JANUARY 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Registered Charity 1066969
[Turn over

Section A (36 marks)

1 The position vector, $\mathbf{r}$, of a particle of mass 4 kg at time t is given by

$$\mathbf{r} = t^2\mathbf{i} + (5t - 2t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors, lengths are in metres and time is in seconds.

(i) Find an expression for the acceleration of the particle. [4]

The particle is subject to a force $\mathbf{F}$ and a force $12\mathbf{j}$ N.

(ii) Find $\mathbf{F}$. [3]

2 Particles of mass 2 kg and 4 kg are attached to the ends X and Y of a light, inextensible string. The string passes round fixed, smooth pulleys at P, Q and R, as shown in Fig. 2. The system is released from rest with the string taut.

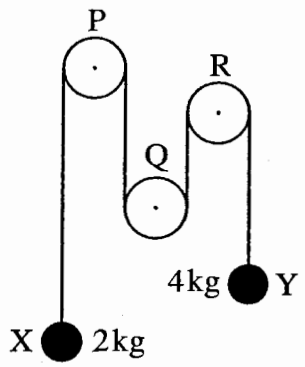


Fig. 2

(i) State what information in the question tells you that

(*A*) the tension is the same throughout the string,

(*B*) the magnitudes of the accelerations of the particles at X and Y are the same. [2]

The tension in the string is T N and the magnitude of the acceleration of the particles is $a\,\mathrm{m\,s^{-2}}$.

(ii) Draw a diagram showing the forces acting at X and a diagram showing the forces acting at Y. [1]

(iii) Write down equations of motion for the particles at X and at Y. Hence calculate the values of T and a. [5]

3 A particle is in equilibrium when acted on by the forces $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix}$, $\begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$, where the units are newtons.

(i) Find the values of x, y and z. [4]

(ii) Calculate the magnitude of $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$. [2]

4 A particle is projected vertically upwards from a point O at $21\,\text{ms}^{-1}$.

(i) Calculate the greatest height reached by the particle. [2]

When this particle is at its highest point, a second particle is projected vertically upwards from O at $15\,\text{ms}^{-1}$.

(ii) Show that the particles collide 1.5 seconds later and determine the height above O at which the collision takes place. [6]

5 A small box B of weight 400 N is held in equilibrium by two light strings AB and BC. The string BC is fixed at C. The end A of string AB is fixed so that AB is at an angle α to the vertical where $\alpha < 60°$. String BC is at 60° to the vertical. This information is shown in Fig. 5.

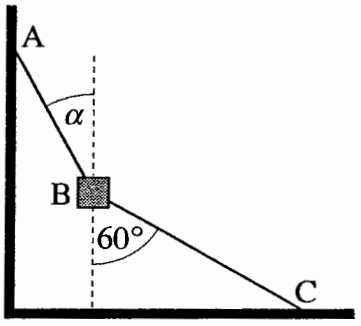


Fig. 5

(i) Draw a labelled diagram showing all the forces acting on the box. [1]

(ii) In one situation string AB is fixed so that $\alpha = 30°$.

By drawing a triangle of forces, or otherwise, calculate the tension in the string BC and the tension in the string AB. [4]

(iii) Show carefully, but briefly, that the box cannot be in equilibrium if $\alpha = 60°$ and BC remains at 60° to the vertical. [2]

[Turn over

Section B (36 marks)

6 In this question take g as $10\,\mathrm{m\,s^{-2}}$.

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.

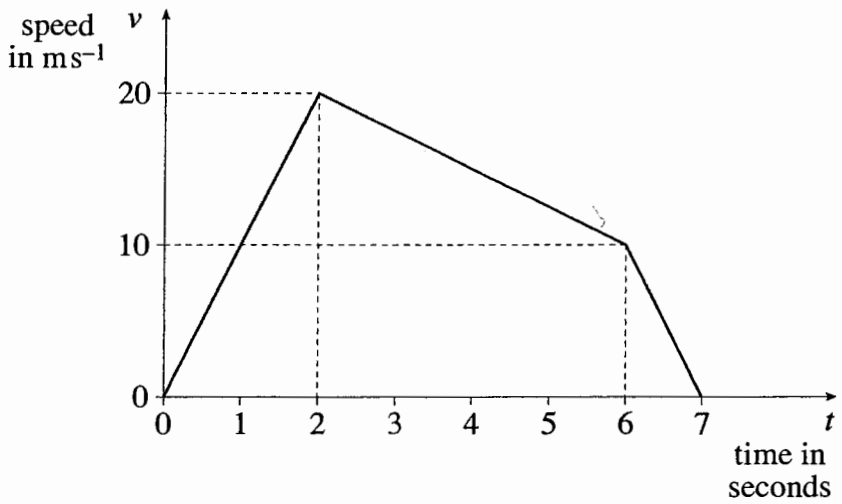


Fig. 6

For this model,

(i) calculate the distance fallen from $t = 0$ to $t = 7$, [3]

(ii) find the acceleration of the ball from $t = 2$ to $t = 6$, specifying the direction, [3]

(iii) obtain an expression in terms of t for the downward speed of the ball from $t = 2$ to $t = 6$, [3]

(iv) state the assumption that has been made about the resistance to motion from $t = 0$ to $t = 2$. [1]

The part of the motion from $t = 2$ to $t = 7$ is now modelled by $v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$.

(v) Verify that v agrees with the values given in Fig. 6 at $t = 2$, $t = 6$ and $t = 7$. [2]

(vi) Calculate the distance fallen from $t = 2$ to $t = 7$ according to this model. [7]

7 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 7. AD is horizontal and BC is parallel to AD.

The stone is projected from A with speed $40\,\mathrm{m\,s^{-1}}$ at 50° to the horizontal.

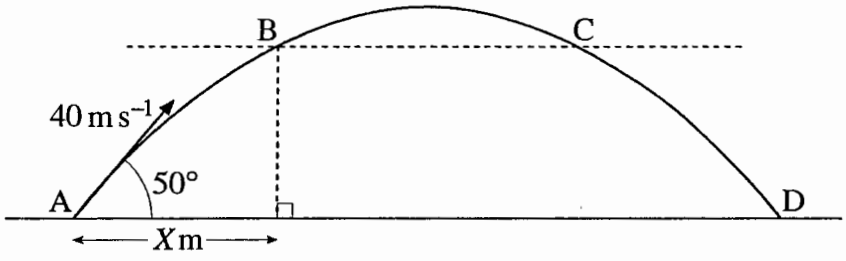


Fig. 7

(i) Write down an expression for the horizontal displacement from A of the stone t seconds after projection. Write down also an expression for the vertical displacement at time t. [3]

(ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. Calculate the range of the stone. [5]

You are given that $X = 30$.

(iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C. [4]

(iv) Calculate the direction of the motion of the stone at C. [5]

Mark Scheme

Paper 4761	Name Mechanics 1	Session Jan	Year 2005	

Solutions and mark scheme

Q 1		mark		
(i)	Differentiate $\mathbf{v} = 2t\,\mathbf{i} + (5 - 4t)\,\mathbf{j}$ Differentiate $\mathbf{a} = 2\,\mathbf{i} - 4\,\mathbf{j}$	M1 A1 M1 F1	At least 1 cpt correct Award for RHS seen Do not award if **i** and **j** lost in **v.** At least 1 cpt correct. FT FT from **their** 2 component **v**	4
(ii)	$\mathbf{F} + 12\,\mathbf{j} = 4(2\,\mathbf{i} - 4\,\mathbf{j})$ $\mathbf{F} = 8\,\mathbf{i} - 28\,\mathbf{j}$	M1 A1 A1	N2L. Allow $\mathbf{F} = mg\,\mathbf{a}$. No extra forces. Allow $12\mathbf{j}$ omitted Allow wrong signs otherwise correct with **their** vector **a**. cao	3
	total	7		

Q 2		mark		
(i) (A) (B)	the pulleys are smooth and the string is light the string is inextensible	E1 E1	Accept only 'the pulley is smooth'.	2
(ii)	Diagrams	B1	All forces present with labels and arrows. Acc not reqd.	1
	For X, N2L upwards $T - 2g = 2a$ For Y, N2L downwards $4g - T = 4a$ Solve for a and T $a = \frac{g}{3}$ (3.27 (3 s. f.)) $T = \frac{8}{3}g$ (26.1 (3 s. f.))	M1 A1 A1 A1 F1	N2L. Allow $F = mga$. All forces present Award for equation for X or Y or combined Any form Any form FT second answer	5
	total	8		

Paper 4761	Name Mechanics 1	Session Jan	Year 2005	

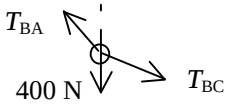
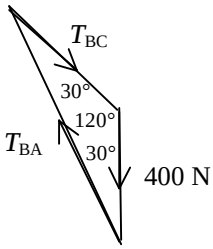
Solutions and mark scheme

Q 3		mark		
(i)	$\begin{pmatrix} x \\ -7 \\ z \end{pmatrix} + \begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Equating components gives x = -9, y = 3, z = 12	M1 A1 A1 A1	[Allow SC 2/4 if 9, -3, -12 obtained]	4
(ii)	We need $\sqrt{5^2 + 4^2 + (-7)^2}$ = $\sqrt{90}$ or 9.48683… so 9.49 (3 s. f.)	M1 A1	Any reasonable accuracy	2
	total	6		

Q 4		mark		
(i)	Height reached by first particle is given by $0 = 21^2 - 2\times 9.8\times s$ so s = 22.5 so 22.5 m	 M1 A1	 Other methods must be complete. Allow $g = \pm 9.8, \pm 10$ Accept with consistent signs	2
(ii)	**Sol (1)** t seconds after second particle projected its height is $15t - 4.9t^2$ and the first particle has height $22.5 - 4.9t^2$ (or $21t - 4.9t^2$) **either** Sub t = 1.5 to show both have same value State height as 11.475 m **or** $15t - 4.9t^2 = 22.5 - 4.9t^2$ giving t = 1.5 and height as 11.475 m	M1 A1 M1 A1 E1 A1 M1 A1	Allow $g = \pm 9.8, \pm 10$ Allow $g = \pm 9.8, \pm 10$ Award only if used correctly (or sub t = 3.64 into $21t - 4.9t^2$ for 1st & t = 1.5 for 2nd) cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained. Both. t shown. Ht cao (to any reasonable accuracy)	
	Sol (2) t seconds after second particle projected its height is $15t - 4.9t^2$ and the first particle has fallen $4.9t^2$ Collide when $15T - 4.9T^2 + 4.9T^2 = 22.5$ so $T = 1.5$ H = $22.5 - 4.9\times 1.5^2 = 11.475$ m	M1 A1 B1 M1 E1 A1	Allow $g = \pm 9.8, \pm 10$ Or other correct method cao. Accept any reasonable accuracy. Don't award if only one correctly used equation obtained.	6
	total	8		

Paper 4761	Name Mechanics 1	Session Jan	Year 2005	

Solutions and mark scheme

Q 5		mark		
(i)		B1	Different labels. All forces present with arrows in correct directions. Condone no angles.	1
(ii)	Using triangle of forces  Triangle isosceles so tension in BC is 400 N Tension in BA is $2\times400\times\cos 30 = 400\sqrt{3}$ N (693 N, (3 s. f.))	M1 B1 A1 F1	Attempt at triangle of forces. Ignore angles and arrows. Accept 90, 60, 30 triangle. Triangle, arrows, labels and angles correct cao FT BC only [If resolution used, M1 for 1 equn; M1 for 2nd equn + attempt to elim; A1; F1. For M marks all forces present but allow $s \leftrightarrow c$ and sign errors. No extra forces. If Lami used: M1 first pair of equations in correct format, condone wrong angles. A1. M1 second pair in correct format, with **correct** angles.F1 FT their first answer if necessary.]	4
(iii)	Resolve at B perpendicular to the line ABC Weight has unbalanced component in this direction	E1 E1	Attempt to argue unbalanced force Complete, convincing argument. [**or** Resolve horiz and establish tensions equal E1 Resolve vert to show inconsistency. E1]	2
	total	7		

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Solutions and mark scheme

Q 6		mark		
(i)	Area under curve $0.5\times2\times20+0.5\times(20+10)\times4+0.5\times10\times1$ = 85 m	M1 B1 A1	Attempt to find any area under curve or use const accn results Any area correct (Accept 20 or 60 or 5 without explanation) cao	3
(ii)	$\frac{20-10}{4}=2.5$ upwards	M1 A1 B1	$\Delta v/\Delta t$ accept ±2.5 Accept – 2.5 downwards (allow direction specified by diagram etc). Accept 'opposite direction to motion'.	3
(iii)	$v=-2.5t+c$ v = 20 when t = 2 v = – 2.5t + 25	M1 M1 A1	Allow **their** a in the form $v=\pm at+c$ or $v=\pm a(t-2)+c$ cao [Allow $v=20-2.5(t-2)$] [Allow 2/3 for different variable to t used, e.g. x. Allow any variable name for speed]	3
(iv)	Falling with negligible resistance	E1	Accept 'zero resistance', or 'no resistance' seen.	1
(v)	$-1.5\times4+9.5\times2+7=20$ $-1.5\times36+9.5\times6+7=10$ $-1.5\times49+9.5\times7+7=0$	E1 E1	One of the results shown All three shown. Be generous about the 'show'.	2
(vi)	$\int_2^7(-1.5t^2+9.5t+7)dt$ $=\left[-0.5t^3+4.75t^2+7t\right]_2^7$ $=\left(-\frac{343}{2}+\frac{19\times49}{4}+49\right)-(-4+19+14)$ = 81.25 m	M1 A1 A1 A1 M1 A1 A1	Limits not required A1 for each term. Limits not required. Condone + c Attempt to use both limits on an integrated expression Correct substitution in **their** expression including subtraction (may be left as an expression). cao.	7
	total	19		

Paper 4761	Name Mechanics 1	Session Jan	Year 2005	

Solutions and mark scheme

Q 7		mark		
(i)	Horiz $(40\cos 50)t$	B1		
	Vert $(40\sin 50)t - 4.9t^2$	M1	Use of $s = ut + 0.5at^2$ with $a = \pm 9.8$ or ± 10. Allow u = 40. Condone s $\leftrightarrow$ c.	
		A1	Any form	3
(ii)	Need $(40\sin 50)t - 4.9t^2 = 0$	M1	Equating **their** y to zero. Allow quadratic y only	
	so $t = \dfrac{40\sin 50}{4.9}$	M1	Dep on 1st M1. Attempt to solve.	
	= 6.2534… so 6.253 s (3 d. p.)	E1	Clearly shown [**or** M1 (allow u = 40 and s $\leftrightarrow$ c) A1 time to greatest height; E1]	
	Range is $(40\cos 50)\times 6.2534...$	M1	Use of **their** horiz expression	
	= 160.78… so 161 m (3 s. f.)	A1	Any reasonable accuracy	5
(iii)	Time AB is given by $(40\cos 50)T = 30$ so T = 1.16679… so 1.17 s	M1 A1	Equating **their** linear x to 30.	
	then **either** By symmetry, time AC is time AD – time AB	M1	Symmetry need not be explicit. Method may be implied. Any valid method using symmetry.	
	so time AC is 6.2534… $-\dfrac{30}{40\cos 50}$ = 5.086…. so 5.09 s (3 s. f.)	A1	cao	
	or height is $(40\sin 50)T - 4.9T^2$ and we need $(40\sin 50)t - 4.9t^2 = (40\sin 50)T - 4.9T^2$ solved for larger root	M1	Complete method to find time to second occasion at that height	
	i.e. solve $4.9t^2 - (40\sin 50)t + 29.08712... = 0$ for larger root giving 5.086…	A1	cao	4
(iv)	$\dot{x} = 40\cos 50$	B1	Must be part of a method using velocities.	
	$\dot{y} = 40\sin 50 - 9.8\times 5.086...$	M1 A1	Use of vert cpt of vel Allow only sign error. FT use of **their** 5.086..	
	Need $\arctan\dfrac{\dot{y}}{\dot{x}}$	M1	May be implied. Accept $\arctan\dfrac{\dot{x}}{\dot{y}}$ but not use of $\dot{y}_0$.	
	So –36.761…° so 36.8° below horizontal (3 s.f.)	A1	Accept ±36.8 or equivalent. Condone direction not clear.	5
	total	17		